

$$A1) a) \quad \boxed{M = 1 \bar{s} = 1 \pi_n^2 \bar{n}}$$

\vec{n}

$$I = \frac{e}{T} \text{ avec } 2\pi n = vT \rightarrow I = \frac{ev}{2\pi n} \quad \Rightarrow \quad M = \frac{evn}{2} \vec{n}$$

$$1) b) \quad \vec{j} = M_p \vec{r} \wedge \vec{v} \quad \| \vec{j} \| = M_p r v \quad \text{car } \vec{v} \perp \vec{r} \text{ pour un mot circ.}$$

$$01.25 \quad 7) c) \text{ Par def } \vec{M} = \gamma \vec{j} \Rightarrow \boxed{\gamma = \frac{M}{j} = \frac{e v r}{2 M_p r v} = \frac{e}{2 M_p}}$$

$$0.25 \quad 1d) \quad \frac{g_{\text{real}}}{g_{\text{Earth}}} = 5,58 \cdot \frac{e}{2 \cdot 10^{-27}} = 5,58 \cdot \frac{1,6 \cdot 10^{-19}}{2 \cdot 1,6 \cdot 10^{-27}} = 2,79 \cdot 10^8 \text{ c. kg}^{-1}$$

$$2) a) E = -M \cdot \vec{B} = -M_B \quad | \text{ aus } M \parallel B$$

$$M_3 = \gamma j_3 \quad \begin{cases} f_+ = \pm \frac{\hbar}{4\pi} B_0 \end{cases} \Rightarrow E_{\pm} = \mp \frac{\gamma \hbar}{4\pi} B_0 \Rightarrow \Delta E = |E_+ - E_-| + \frac{\gamma \hbar}{2\pi} B_0 \quad \text{proportionnelle à } B_0$$

$$\text{Def: } \Delta E = \frac{\hbar \omega_0}{2\pi} \Rightarrow \omega_0 = \frac{2\pi \Delta E}{\hbar} = \boxed{Y B_0}$$

A.1.

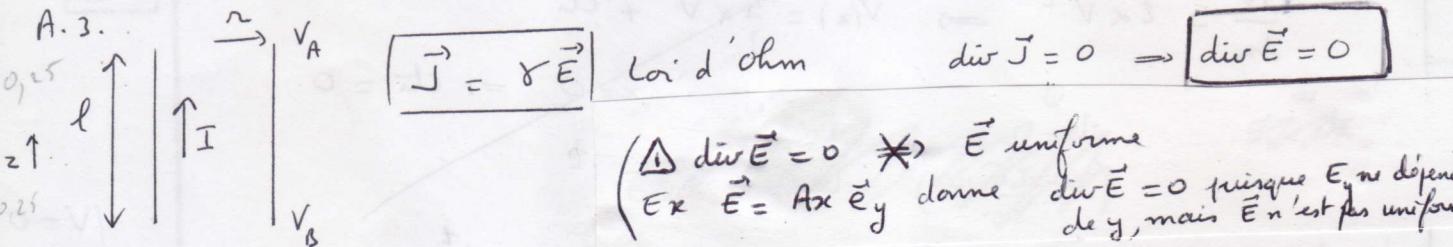
$$\operatorname{div} \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

0,25

A.2. rég. stationnaire $\Rightarrow \rho \neq \rho(t) \Rightarrow \operatorname{div} \vec{J} = 0$ (\Rightarrow loi des noeuds)

0,25

A.3.



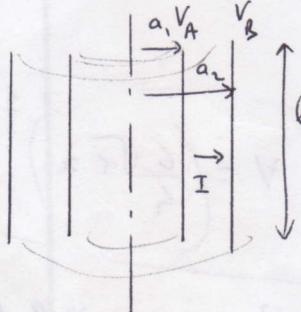
0,25

$$dV = -\vec{E} \cdot d\vec{r}$$

$$V_A - V_B = -E_z \cdot l$$

$$\text{Def: } V_B - V_A = RI$$

A.5



$$dV = -\vec{E} \cdot d\vec{r}$$

$$\text{I radial} \Rightarrow J \text{ radial : } I = J \times l \times 2\pi r \Rightarrow J(r) = \frac{I}{2\pi r l}$$

$$\vec{J} = \gamma \vec{E} \Rightarrow \vec{E} = \frac{\vec{J}}{\gamma} \rightarrow E(r) = \frac{I}{2\pi r l \gamma}$$

$$dV = -\vec{E} \cdot d\vec{r} = -\frac{I}{2\pi r l \gamma} \vec{r} \cdot d\vec{r} = -\frac{I}{2\pi r^2 l \gamma} r dr$$

$$\Rightarrow dV = -\frac{I}{2\pi r l \gamma} dr$$

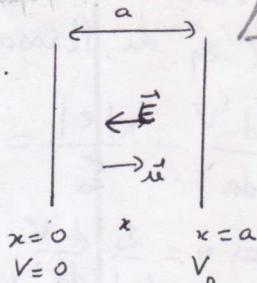
$$\text{Def: } V_B - V_A = RI$$

$$V_B - V_A = -\frac{I}{2\pi l \gamma} \ln \frac{a_2}{a_1} \Rightarrow R = \frac{\ln \frac{a_2}{a_1}}{2\pi l \gamma}$$

1,5

(pas bonnes pas expliquées)

B.1.

 $\Delta: E \neq \text{uniforme}$

$$\text{Th. } E_c: d\left(\frac{1}{2}mu^2\right) = \delta W_F = -|e|E dx = |e| dV$$

$$\text{Intégrer: } \Delta\left(\frac{1}{2}mu^2\right) = |e| \Delta V$$

$(\text{à } x=0 \text{ et } V=0)$

$$u = \sqrt{\frac{2eV}{m}}$$

0,5

$$B.2. \quad \vec{J} = n(-e) \vec{u} \quad \vec{u} = u \vec{e}_x \quad \Rightarrow \quad \boxed{\vec{J} = -neu \vec{e}_x}$$

B.3. Eq Poisson (rég. stationnaire, pas de propagation) $\Delta V + \frac{\rho}{\epsilon_0} = 0$

$$\text{ici } \rho = -ne$$

$$\boxed{\Delta V - \frac{ne}{\epsilon_0} = 0}$$

$$\text{ou } \alpha = \frac{|J|}{\epsilon_0} \sqrt{\frac{m}{2e}}$$

0,5

$$\frac{d^2V}{dx^2} - \frac{ne}{\epsilon_0} = 0$$

$$n = -\frac{J_x}{eue} = -\frac{J_x}{e} \sqrt{\frac{m}{2eV}}$$

$$\Rightarrow \frac{d^2V}{dx^2} + \frac{J_x \sqrt{\frac{m}{2e}}}{\epsilon_0} V^{-\frac{1}{2}} = 0$$

$$\alpha = \frac{-J_x}{\epsilon_0} \sqrt{\frac{m}{2e}}$$

$\alpha > 0$

1,5

$$B.S. \quad \frac{d^2V}{dx^2} = \alpha V^{-\frac{1}{2}} \quad (2)$$

$$V''_+ V' = \alpha V^{-\frac{1}{2}} V' \quad \text{ou} \quad V' = \frac{dV}{dx}$$

$$\text{Intégrer: } \left[\frac{1}{2} V'^2 \right] = \left[2\alpha V^{\frac{1}{2}} \right]$$

$$\text{à } E = -V' = 0 \quad \text{à } x=0 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow \frac{1}{2} V'^2 = 2\alpha V^{\frac{1}{2}}$$

$$V' = 4 \times V^{\frac{1}{2}}$$

$$V' = 2\sqrt{\alpha} V^{\frac{1}{4}}$$

$$\frac{dV}{V^{\frac{1}{4}}} = 2\sqrt{\alpha} dx \Rightarrow \left| \int V^{\frac{3}{4}} \right| = 2\sqrt{\alpha} x + C$$

$$V^{\frac{3}{4}} = \frac{3}{2} \sqrt{\alpha} x + C$$

$$V=0 \quad \text{à } x=0 \Rightarrow C=0$$

$$2 \quad \boxed{V = \left(\frac{3}{2} \right)^{\frac{4}{3}} \alpha^{\frac{2}{3}} x^{\frac{4}{3}}}$$

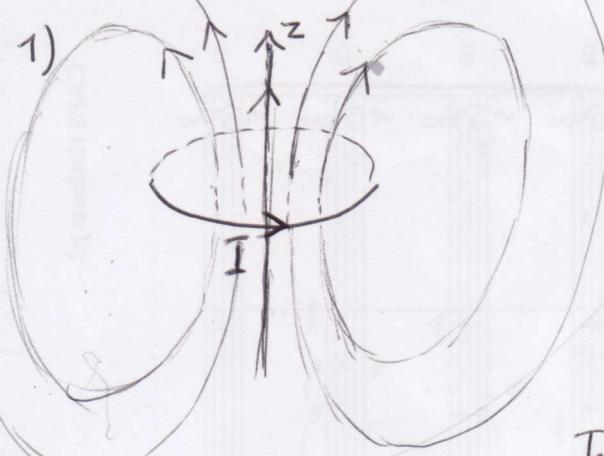
2

$$B.5. \quad \alpha = -\frac{1}{e} \sqrt{\frac{m}{2e}} \quad (J < 0)$$

$$B.S. \quad V_0 = \left(\frac{3}{2} \right)^{\frac{4}{3}} \alpha^{\frac{2}{3}} a^{\frac{4}{3}}$$

$$\Rightarrow V_0^{\frac{2}{3}} \left(\frac{2}{3} \right)^2 a^{-2} = -\frac{1}{e_0} \sqrt{\frac{m}{2e}}$$

$$\boxed{V_0 = \left(\frac{3}{2} \right)^{\frac{4}{3}} a^{\frac{4}{3}} \left(\frac{2}{e_0} \right)^{\frac{1}{3}} \left(\frac{m}{2e} \right)^{\frac{1}{3}}} \quad (J < 0)$$



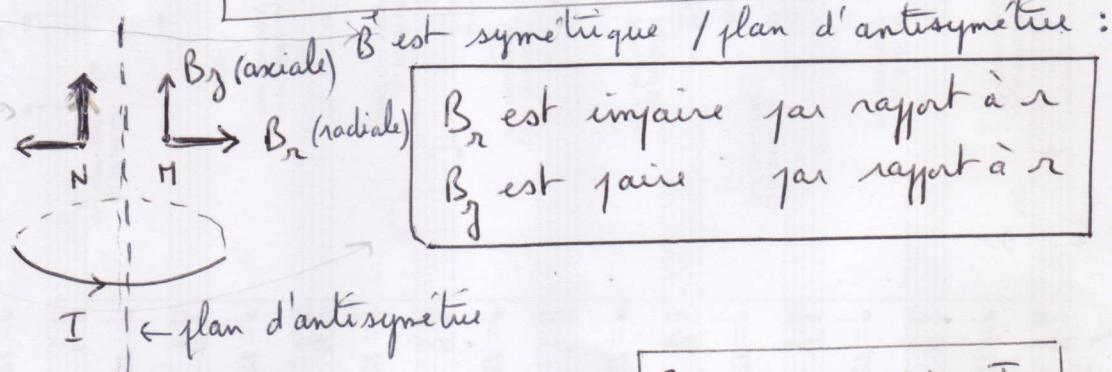
Symétrie: le plan passant par l'axe et un point Π est d'antisymétrique pour le courant I

$\rightarrow \vec{B} \parallel$ ce plan
donc $\vec{B} \in (\vec{u}_x, \vec{u}_y)$

Invariance, I est invariant selon θ
donc $B(r, \theta, z)$

Résumé:

$$\vec{B} = B_r(r, z) \vec{u}_r + B_\theta(r, z) \vec{u}_\theta$$



2) Expressions avec B_r impaire : $B_r(r, z) = \alpha(z) r I$
 B_θ paire : $B_\theta(r, z) = f(z) I$

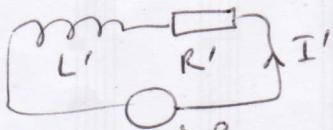
Eq^ locale de cons^ de $\Phi(B)$: $\operatorname{div} \vec{B} = 0$
sat $\frac{1}{r} \frac{\partial (r B_r)}{\partial r} + \frac{\partial B_\theta}{\partial z} = 0 \rightarrow \frac{1}{r} \alpha'(z) + f'(z) = 0$
 $\rightarrow \alpha(z) = -\frac{1}{2} f'(z)$

3) $\Phi(\vec{B}, c') = \frac{B_{(n=0, \theta)}}{2} \pi b^2 = f(z) I \pi b^2$

4) Loi de Faraday: c'est le siège d'une tension induite

$$e = -\frac{\partial \Phi}{\partial t} = -f(z) \pi b^2 \frac{dI}{dt}$$

Schéma électrique de la spire (c')



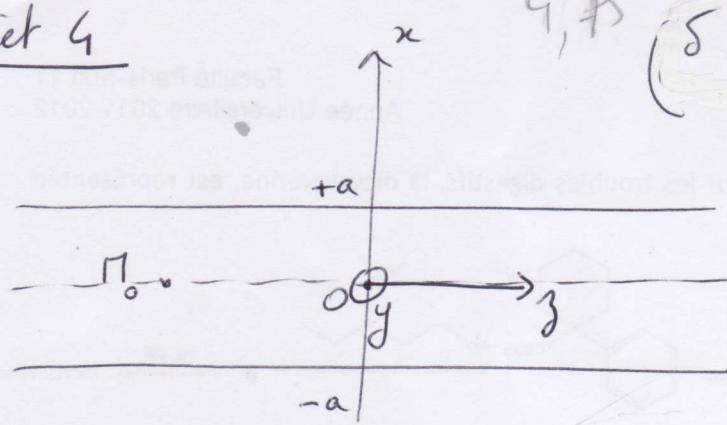
$$e = L' \frac{dI'}{dt} + RI'$$

$$-\frac{f(z) \pi b^2}{2} \frac{dI}{dt} = L' \frac{dI'}{dt} + RI'$$

Supraconducteur: $|R'|=0 \rightarrow -f(z) \pi b^2 \frac{dI}{dt} = L' \frac{dI'}{dt}$

Intégrer: $-f(z) \pi b^2 I_0 = L' I'_0$

Sujet 4



4,75

($\delta \tau$ longueur si j_0 est une densité de courant)

$$\vec{j} = -j_0 \frac{x}{5^2} \hat{e}_y$$

j est symétrique / Oyy

- a) Le plan Oyy est de symétrie pour le courant
Le plan Oxy

$\Rightarrow \vec{B}$ perpendiculaire à ces 2 plans donc $B(n_0) = 0$

En Π le plan (M_{xy}) est de symétrie
donc $\vec{B}(n) \parallel \hat{e}_z$

0,5

0,25

0,25

Invariance de la répartition de courants selon y et z donc $B(x)$

Résumé: $\vec{B}(n) = B(x) \hat{e}_z$

b) $\nabla \cdot \vec{B} = \mu_0 \vec{j}$

$$\begin{matrix} \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial z} & B \end{matrix} = \begin{cases} \frac{\partial B}{\partial y} = 0 \\ -\frac{\partial B}{\partial x} = \mu_0 j_0 \end{cases} \Rightarrow \begin{cases} 0 \\ \mu_0 j_0 \\ 0 \end{cases}$$

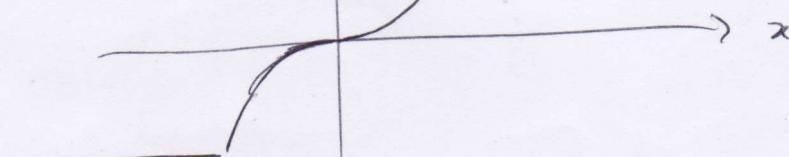
$$-\frac{\partial B}{\partial x} = \mu_0 j_0$$

Si $x \in [0; a]$ alors $-\frac{\partial B}{\partial x} = -\mu_0 j_0 \frac{x}{5^2} \rightarrow B = \mu_0 j_0 \frac{x^2}{35^2} + C_1$

Si $x > a$ alors $-\frac{\partial B}{\partial x} = 0 \rightarrow B = C_2$

Sachant que $B(0) = 0$ $C_1 = 0$ d'où par continuité $C_2 = \mu_0 j_0 \frac{a^3}{35^2}$

$$B \underset{x=0}{\overset{x=a}{\sim}} \mu_0 j_0 \frac{a^3}{35^2}$$



c) $\delta \vec{F}_{\text{Laplace}} = \vec{j} \delta \tau \wedge \vec{B} = -\frac{1}{\mu_0} \frac{\partial B}{\partial x} \hat{e}_y \delta x \delta y \delta z \vec{B} \hat{e}_z = -\frac{1}{\mu_0} \delta y \delta z \vec{B} dB(\hat{e}_x)$

$$\vec{F}_{\text{Laplace}} = -\frac{1}{2\mu_0} [B^2]_0^a \delta y \delta z \hat{e}_x = -\frac{1}{2\mu_0} (B(a)^2 - B(0)^2) \delta y \delta z \hat{e}_x$$

$$= -\frac{1}{2\mu_0} \left(\mu_0 j_0 \frac{a^3}{35^2} \right)^2 \delta y \delta z \hat{e}_x$$

1) a) Le conducteur est ∞ selon y et z donc \vec{E} ne dépend pas de y ni de z .
 $\Rightarrow \vec{E}$ ne peut dépendre que de x

$$\Delta \vec{E} \doteq \Delta E_x \vec{e}_x + \Delta E_y \vec{e}_y + \Delta E_z \vec{e}_z = \frac{\partial^2 E_x}{\partial x^2} \vec{e}_x + \frac{\partial^2 E_y}{\partial x^2} \vec{e}_y + \frac{\partial^2 E_z}{\partial x^2} \vec{e}_z = \frac{\partial^2 \vec{E}}{\partial x^2}$$

$$\Delta \vec{E} - \frac{\vec{E}}{\lambda_D} = \vec{0} \Rightarrow \frac{\partial^2 \vec{E}}{\partial x^2} - \frac{\vec{E}}{\lambda_D} = \vec{0} \Rightarrow \vec{E} = \vec{E}_1 e^{\frac{x}{\lambda_D}} + \vec{E}_2 e^{-\frac{x}{\lambda_D}}$$

$$\text{or } \vec{E} \xrightarrow[x \rightarrow \infty]{} \vec{0} \text{ donc } \vec{E}_1 = \vec{0} \text{ d'où } \vec{E} = \vec{E}_2 e^{-\frac{x}{\lambda_D}}$$

$$\text{continuité de } \vec{E} \text{ à } x=0 \text{ (traversée d'une densité volumique)} : \vec{E}_L = \vec{E}_0$$

$$\Rightarrow \vec{E} = E_0 e^{-\frac{x}{\lambda_D}} \vec{e}_x$$

$$\text{div } \vec{E} = \rho \text{ et } \text{div } \vec{E} \doteq \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\partial E_x}{\partial x} \Rightarrow \frac{\partial E_x}{\partial x} = \rho \Rightarrow -\frac{E_0}{\lambda_D} e^{-\frac{x}{\lambda_D}} = \rho$$

$$\Rightarrow \rho = -\frac{E_0}{\lambda_D} e^{-\frac{x}{\lambda_D}} ; \text{ en } x=0$$

$$\left. \rho_{\text{surface}} = -\frac{E_0}{\lambda_D} \right\rangle \rho_0$$

$$1) b) \tau dS = \int_0^\infty \rho dS dx = \int_0^\infty -\frac{E_0}{\lambda_D} e^{-\frac{x}{\lambda_D}} dS dx \Rightarrow \tau = \int_0^\infty \rho dx = \int_0^\infty -\frac{E_0}{\lambda_D} e^{-\frac{x}{\lambda_D}} dx$$

($dS = dy dz$)

$$\tau = -\frac{E_0}{\lambda_D} \left[\rho \right]_0^\infty$$

$$\left. \tau = -E_0 = \rho \lambda_D \right\rangle$$

2) a)

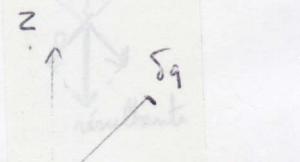
Tous les plans passant par O et d'axe Oz sont de symétrie

$$\Rightarrow \vec{E}_m(0) \parallel \vec{Oz}$$

$$\vec{E}_m = E_m \vec{e}_z = \int_{\theta=0}^{\pi} \frac{2\pi R \sin \theta \, R d\theta}{4\pi \epsilon_0 R^2} \times \cos \theta (-\vec{e}_z)$$

$$\begin{aligned} (\tau = \tau_0 \cos \theta) \\ = -\frac{\tau_0}{2\epsilon_0} \int_0^{\pi} \sin \theta \cos^2 \theta d\theta \vec{e}_z = -\frac{\tau_0}{2\epsilon_0} \left[-\frac{\cos^3 \theta}{3} \right]_0^{\pi} \vec{e}_z \end{aligned}$$

SE $\vec{0}$ projection de $\delta \vec{E}$ (négative)



projection de $\delta \vec{E}$ (négative)

$$\Rightarrow \vec{E}_m = -\frac{\tau_0}{3\epsilon_0} \vec{e}_z$$

$$\Rightarrow \vec{E} = \vec{E}_m + \vec{E}_a = \left(E_a - \frac{\tau_0}{3\epsilon_0} \right) \vec{e}_z$$

$$b) \delta \vec{P} \doteq \delta q \vec{AB} \quad \text{ou}$$



$$\begin{aligned} B \delta q &= \tau_0 \cos \theta \delta S \\ -\delta q &= \tau_0 \cos(\theta - \pi) \delta S \end{aligned}$$

$$\text{avec } \delta S = R d\theta \times R \sin \theta d\varphi \text{ et } AB = 2R \cos \theta$$

$$\text{donc } \delta \vec{P} = \tau_0 \cos \theta R^2 \sin \theta d\theta d\varphi \times 2R \cos \theta \vec{e}_z$$

$$\left. \delta \vec{P} = 2\tau_0 R^3 \sin \theta \cos^2 \theta d\theta d\varphi \vec{e}_z \right\rangle$$

$$\vec{P} = \int \delta \vec{P} = 2\sigma_0 R^3 \int_0^{\pi} \sin \theta \cos^2 \theta d\theta \int_0^{2\pi} dy \vec{e}_z$$

$$2 \quad \boxed{\vec{P} = 2\sigma_0 R^3 \left[-\frac{\cos^3 \theta}{3} \right]_0^{\pi} 2\pi \vec{e}_z = \frac{4\pi R^3 \sigma_0}{3} \vec{e}_z}$$

$$\vec{E} = \vec{E}_a - \frac{\vec{P}}{3\epsilon_0}$$

$$\vec{P} = \frac{\vec{I}}{\frac{4\pi R^3}{3}} = \sigma_0 \vec{e}_z$$